ISSN: 2277-9655
Impact Factor: 1.852

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Estimation in Step Stress Partially Accelerated Life Tests for Rayleigh Distribution Using Type-I Censoring

Shashi Saxena[*1], Shazia Zarrin[2], Arif-ul Islam[3]
[*1,2,3] Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh-202002, India
sshashisaxena@gmail.com

Abstract

In this study a step stress partially accelerated life test (SSPALT) using type-I censoring is obtained for Rayleigh distribution. This distribution has been found appropriate for high reliability components. Maximum Likelihood (ML) Estimation is used to estimate the parameters of SSPALT model. Confidence intervals for the model parameters are constructed. An iterative procedure is also used to obtain the estimators numerically.

Keywords : Acceleration factor; Step stress; Fisher Information matrix; generalized asymptotic variance; Optimum test plan; Maximum likelihood estimation; Confidence intervals.

Introduction

Due to the rapid advances in technology, development of highly sophisticated products, intense global competition, and increasing customer expectations, the reliability of the products in recent era have become very high. As in life testing experiments the failure time data is used to obtain the product life characteristics under normal operating conditions, therefore, such life data has become very difficult to obtain as a result of the great reliability of today's products and hence under normal operating conditions, as products usually last long, the corresponding life-tests become very time consuming and expensive. In these cases, accelerated tests can be applied to reduce the experimental time and hence the cost. Accelerated life testing (ALT) is a quick way to obtain information about the life distribution of a material, component or product. In ALT items are subjected to conditions that are more severe than the normal ones, which yields shorter life but, hopefully, do not change the failure mechanisms. Failure information collected under this severe test stresses can be extrapolated to obtain an estimate of lifetime under use condition based on some life-stress relationship. Such a way of testing reduces time and cost.

ALTs, generally deal with three types of stress loadings: constant stress, step stress and progressive stress. Constant stress is the most common type of stress loading, in which every item is tested under a constant level of the stress, which is higher than normal level. In this kind of testing, we may have several stress levels, which are applied for different groups of the tested items. This means that every item is subjected to only one stress level until the item fails or the test is stopped for other reasons. If the stress level of the test is not high enough, many of the tested items will not fail during the available time and one has to be prepared to handle a lot of censored data. To avoid this problem, step-stress testing can be applied, in which, all items are first subjected to a specified constant stress for a specified period of time. Items that do not fail will be subjected to a higher level of stress for another specified time. The level of stress is increased step by step until all items have failed or the test stops for other reasons. Progressive-stress loading is quite like the step stress testing with the difference that the stress level increases continuously.

Failure data obtained from ALT can be divided into two categories: complete (all failure data are available) or censored (some of failure data are missing). Complete data consist of the exact failure time of test units, which means that the failure time of each sample unit is observed or known. In many cases when life data are analyzed, all units in the sample may not fail. This type of data is called censored or incomplete data. See for more details, Bagdonavicius and Nikulin (2002), Meeker and Escobar (1998), Nelson (1980, 1990), Mann and Singpurwalla (1983).

The fundamental assumption in ALT is that the mathematical model relating the lifetime of the unit and the stress should be known or can be assumed. In some cases, this kind of life-stress relationships are not known and also cannot be assumed, i.e. ALT data cannot be extrapolated to use condition. So, in such cases, partially accelerated life tests (PALT) is a more suitable test to be performed for which tested units are subjected to both normal and accelerated conditions. According to Nelson

(1990), the stress can be applied in various ways. One way to accelerate failure is step-stress, which increases the stress applied to test product in a specified discrete sequence. Step-stress partially accelerated life test (SS-PALT) is used to get quick information for the lifetime of product with high reliability; specially, when the mathematical model related to test conditions of mean lifetime of the product is unknown and cannot be assumed.

For an overview of SS-PALT, there is amount of literature on designing SS- PALT. Goel (1971) considered the estimation problem of the acceleration factor using both maximum likelihood and Bayesian methods for items having exponential and uniform distributions. DeGroot and Goel (1979) estimated the parameters of the exponential distribution and acceleration factor in SS-PALT using Bayesian approach, with different loss functions. Also, Bhattacharyya and Soejoeti (1989) estimated the parameters of the Weibull distribution and acceleration factor using maximum likelihood method. Bai and Chung (1992) estimated the scale parameter and acceleration factor for exponential distribution under type I censored sample using maximum likelihood method.

Attia et al. (1996) considered the maximum likelihood method for estimating the acceleration factor and the parameters of Weibull distribution in SS-PALT under type I censoring. Abdel-Ghaly et al. (1997) used Bayesian approach for estimating the parameters of Weibull distribution with known shape parameter. They studied the estimation problem in SS-PALT under both type I and type II censored data. Abdel-Ghani (1998) considered the estimation problem of the parameters of Weibull distribution and the acceleration factor for both SS-PALT and constant-stress PALT. Maximum likelihood and Bayesian methods under type I and type II censored data are applied in this study. Abdel-Ghaly et al. (2002) studied the estimation problem of the acceleration factor and the parameters of Weibull distribution in SS-PALT using maximum likelihood method using both type I and type II censoring.

In this study the maximum likelihood method has been used for estimating the acceleration factor and the model parameters. The lifetime distribution of the test item is assumed to be Rayleigh distribution and the test is conducted for SS-PALT under type I censored sample. The performance of the obtained estimators is investigated in terms of relative absolute bias, mean square error and the relative error. Moreover, the 95% and 99% confidence intervals of the estimators have been obtained.

The Model

The lifetimes of the test items are assumed to follow a Rayleigh distribution. The probability density function of the Rayleigh distribution is given by

$$f(t) = \frac{t}{\theta^2}\exp\left(-\frac{t^2}{2\theta^2}\right), 0 \le t < \infty,\ \theta > 0 \tag{1}$$

where, θ is the scale parameter.

And the cumulative distribution function is given by

$$F(t) = 1 - \exp\left(-\frac{t^2}{2\theta^2}\right), 0 \le t < \infty,\ \theta > 0 \tag{2}$$

The reliability function of the Rayleigh distribution takes the form

$$R(t) = \exp\left(-\frac{t^2}{2\theta^2}\right), \tag{3}$$

and the corresponding hazard rate is given by

$$h(t) = \frac{t}{\theta^2}$$

The Rayleigh distribution has played an important role in modeling the lifetime of random phenomena. It arises in many areas of applications, including reliability, life testing and survival analysis. Weibull distribution converts into Rayleigh distribution when shape parameter takes the value 2. Rayleigh distribution is frequently used to model wave heights in oceanography, and in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year and a wind turbine sites. The distance from one individual to its nearest neighbor when the spatial pattern is generated by Poisson distribution follows a Rayleigh distribution. In communication theory, Rayleigh distribution is used to model scattered signals that reach a receiver by multiple paths. Depending on the density of scatter, the signal will display different fading characteristics. Rayleigh distribution is used to model dense scatter.

While conducting a step-stress PALT, firstly, all of n units are tested under normal use condition, and if the unit does not fail for a pre-fixed time τ, then it is run at accelerated condition until failure. The effect of this change from normal to accelerated condition is to multiply the remaining lifetime of the item by the inverse of the acceleration factor β, say β^{-1}. Acceleration factor is taken to be the ratio of the hazard rate at accelerated condition to that at normal use condition, generally $\beta > 1$. Here, in this case, moving from lower

stress level to the higher stress level will shorten the life of the test item. Thus the total lifetime of the test item, denoted by Y, passes through two stages, first through the normal use conditions and the second through the accelerated conditions. Interested readers may go through Ismail (2006) and Abd- Elfattah et al. (2008).
The total lifetime of an item in SSPALT is given by

$$Y=\begin{cases} T & if\ T\le\tau \\ \tau+\beta^{-1}(T-\tau) & if\ T>1 \end{cases}, \qquad (4)$$

where, T is the lifetime of an item at use condition, τ is the stress change time and β is the acceleration factor. This model is called the tampered random variable (TRV) model. It was proposed by DeGroot and Goel (1979).
Assume that the lifetime of the test item follows a Rayleigh distribution with θ as a scale parameter. Then, the pdf of total lifetime Y of an item is given by

$$f(y)=\begin{cases} 0 & if\ \ y\le 0 \\ f_1(y) & if\ \ 0<y\le\tau \\ f_2(y) & if\ \ y>\tau \end{cases}$$

(5)

Where, $f_1(y)=\dfrac{y}{\theta^2}\exp\left(-\dfrac{y^2}{2\theta^2}\right)$, $\theta>0$, is the equivalent form to (1), and

$$f_2(y)=\frac{\beta[\tau+\beta(y-\tau)]}{\theta^2}\exp\left(-\frac{[\tau+\beta(y-\tau)]^2}{2\theta^2}\right),$$

$\theta>0$, $\beta>1$, is obtained by the transformation variable technique using (1) and (4).

Maximum Likelihood Estimation

The maximum likelihood estimation is one of the most important and widely used methods in statistics. It is commonly used for the most theoretical model and kinds of censored data. The idea behind the maximum likelihood parameter estimation is to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also the MLEs have the desirable properties of being consistent and asymptotically normal for large samples.

Point Estimation

In type I censoring the test terminates when the censoring time η is reached. The observed values of the total lifetime Y are

$$y_{(1)}<\ldots<y_{(n_u)}\le\tau<y_{(n_u+1)}<\ldots<y_{(n_u+n_a)}\le\eta,$$

where n_u and n_a are the number of items failed at normal conditions and accelerated conditions respectively. Let δ_{1i} and δ_{2i} be the indicator functions (**Indicator Function** is a function defined on a set S that indicates membership of an element in a subset A of S, having the value 1 for all elements of A and the value 0 for all elements of S not in A.), such that

$$\delta_{1i}=\begin{cases} 1 & y_i\le\tau \\ 0 & otherwise \end{cases} \qquad i=1,2,\ldots,n$$

and

$$\delta_{2i}=\begin{cases} 1 & \tau<y_i\le\eta \\ 0 & otherwise \end{cases} \qquad j=1,2,\ldots,n$$

The lifetimes $y_1,\ldots,y_n$ of n items are independent and identically distributed random variables, and then their likelihood function is given by

$$L(y;\theta,\beta)=\prod_{i=1}^{n}[f_1(y_i)]^{\delta_{1i}}\times[f_2(y_i)]^{\delta_{2i}}\times[R(\eta)]^{\bar{\delta}_{1i}\bar{\delta}_{2i}} \qquad (6)$$

By substituting the respective values we get

$$L(y;\theta,\beta)=\prod_{i=1}^{n}\left[\frac{y_i}{\theta^2}\exp\left(-\frac{y_i^{\,2}}{2\theta^2}\right)\right]^{\delta_{1i}}\left[\frac{\beta[\tau+\beta(y_i-\tau)]}{\theta^2}\exp\left(-\frac{[\tau+\beta(y_i-\tau)]^2}{2\theta^2}\right)\right]^{\delta_{2i}}$$

$$\left[\exp\left(-\frac{[\tau+\beta(\eta-\tau)]^2}{2\theta^2}\right)\right]^{\bar{\delta}_{1i}\bar{\delta}_{2i}}$$

Where, $\bar{\delta}_{1i}=1-\delta_{1i}$ and $\bar{\delta}_{2i}=1-\delta_{2i}$.

It is usually easier to maximize the natural logarithm of the likelihood function rather than the likelihood function itself. Therefore, the logarithm of the likelihood function is

$$\log L=\sum_{i=1}^{n}\left[\delta_{1i}\left\{\log y_i-2\log\theta-\frac{y_i^2}{2\theta^2}\right\}\right.$$

$$+\delta_{2i}\left\{\log\beta+\log[\tau+\beta(y_i-\tau)]-2\log\theta-\frac{[\tau+\beta(y_i-\tau)]^2}{2\theta^2}\right\}$$

$$\left.-\bar{\delta}_{1i}\bar{\delta}_{2i}\left(\frac{[\tau+\beta(\eta-\tau)]^2}{2\theta^2}\right)\right]$$

$$\log L = -2n_0 \log\theta + \sum_{i=1}^{n} \delta_{1i} \log y_i - \sum_{i=1}^{n} \frac{\delta_{1i} y_i^2}{2\theta^2} + n_a \log\beta + \sum_{i=1}^{n} \delta_{2i} \log[\tau + \beta(y_i - \tau)]$$

$$-\sum_{i=1}^{n} \delta_{2i} \frac{[\tau+\beta(y_i-\tau)]^2}{2\theta^2} - (n-n_0)\left(\frac{[\tau+\beta(\eta-\tau)]^2}{2\theta^2}\right)$$

$\sum_{i=1}^{n} \bar{\delta}_{1i}\bar{\delta}_{2i} = n - n_u - n_a$ and $n_0 = n_u + n_a$.

Maximum likelihood estimators of θ and β are solutions to the system of equations obtained by letting the first partial derivatives of the total log likelihood be zero with respect to θ and β , respectively. Therefore, the system of equations is as follows:

$$\frac{\partial \log L}{\partial\theta} = -\frac{2n_0}{\theta} + \sum_{i=1}^{n} \frac{\delta_{1i} y_i^2}{\theta^3} + \sum_{i=1}^{n} \delta_{2i} \frac{[\tau+\beta(y_i-\tau)]^2}{\theta^3} + (n-n_0)\left(\frac{[\tau+\beta(\eta-\tau)]^2}{\theta^3}\right)$$
(7)

$$\frac{\partial \log L}{\partial\beta} = \frac{n_a}{\beta} + \sum_{i=1}^{n} \frac{\delta_{2i}(y_i-\tau)}{[\tau+\beta(y_i-\tau)]} - \frac{1}{\theta^2}\sum_{i=1}^{n} \delta_{2i}[\tau+\beta(y_i-\tau)](y_i-\tau)$$

$$+\frac{(n-n_0)(\eta-\tau)[\tau+\beta(\eta-\tau)]}{\theta^2} \qquad (8)$$

Since the closed form solution to nonlinear equations (7) and (8) are very hard to obtain. An iterative procedure is applied to solve these equations numerically. Newton-Raphson method is applied for simultaneously solving the nonlinear equations to obtain θ and β.

The asymptotic variance-covariance matrix of β and θ is obtained by numerically inverting the Fisher-information matrix composed of the negative second order derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. The asymptotic Fisher-information matrix can be written as:

$$F = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial\theta^2} & -\frac{\partial^2 \ln L}{\partial\beta\partial\theta} \\ -\frac{\partial^2 \ln L}{\partial\theta\partial\beta} & -\frac{\partial^2 \ln L}{\partial\beta^2} \end{bmatrix}$$

The elements of the above information matrix can be expressed by the following equations:

$$\frac{\partial^2 \log L}{\partial\theta^2} = \frac{2n_0}{\theta^2} - \frac{3}{\theta^4}\left[\sum_{i=1}^{n} \delta_{1i} y_i^2 + \sum_{i=1}^{n} \delta_{2i}[\tau+\beta(y_i-\tau)]^2 + (n-n_0)[\tau+\beta(\eta-\tau)]^2\right]$$

$$\frac{\partial^2 \log L}{\partial\beta^2} = -\frac{n_a}{\beta^2} - \sum_{i=1}^{n} \frac{\delta_{2i}(y_i-\tau)^2}{[\tau+\beta(y_i-\tau)]^2} - \frac{1}{\theta^2}\left[\sum_{i=1}^{n} \delta_{2i}(y_i-\tau)^2 + (n-n_0)(\eta-\tau)^2\right]$$

$$\frac{\partial^2 \log L}{\partial\theta\partial\beta} = \frac{2}{\theta^3}\left[\sum_{i=1}^{n} \delta_{2i}[\tau+\beta(y_i-\tau)](y_i-\tau) + (n-n_0)[\tau+\beta(\eta-\tau)](\eta-\tau)\right]$$

$$\frac{\partial^2 \log L}{\partial\beta\partial\theta} = \frac{2}{\theta^3}\left[\sum_{i=1}^{n} \delta_{2i}[\tau+\beta(y_i-\tau)](y_i-\tau) + (n-n_0)[\tau+\beta(\eta-\tau)](\eta-\tau)\right]$$

Interval Estimates

If $L_\varepsilon = L_\varepsilon(y_1,\ldots,y_n)$ and $U_\varepsilon = U_\varepsilon(y_1,\ldots,y_n)$ are functions of the sample data $y_1,\ldots,y_n$, then a confidence interval for a population parameter ε is given by

$$p[L_\varepsilon \le \varepsilon \le U_\varepsilon] = \gamma \qquad (9)$$

where, L_ε and U_ε are the lower and upper confidence limits which enclose ε with probability γ . The interval $[L_\varepsilon, U_\varepsilon]$ is called a two sided $100\gamma\%$ confidence interval for ε.

For large sample size, the MLEs, under appropriate regularity conditions, are consistent and asymptotically normally distributed.

Therefore, the two sided approximate $100\gamma\%$ confidence limits for the MLE $\hat{\varepsilon}$ of a population parameter ε can be constructed, such that

$$p[-z \le \frac{\hat{\varepsilon}-\varepsilon}{\sigma(\hat{\varepsilon})} \le z] = \gamma \qquad (10)$$

where, z is the $\left[\frac{100(1-\gamma)}{2}\right]$ standard normal percentile. Therefore, the two sided approximate $100\gamma\%$ confidence limits for a population parameter ε can be obtained such that

$$p[\varepsilon - z\sigma(\hat{\varepsilon}) \le \hat{\varepsilon} \le \varepsilon + z\sigma(\hat{\varepsilon})] \cong \gamma \qquad (11)$$

Then, the two sided approximate confidence limits for β and θ will be constructed using (11) with confidence levels 95% and 99%.

Simulation Studies

First a random sample is generated from Rayleigh distribution for different sample sizes $n = 100, 200, ..., 500$. The values of parameters are chosen to be $(\theta = 4.50, 4.30,\ \beta = 0.30,\ 0.70)$. The censoring time τ at the normal condition is 2 and

ISSN: 2277-9655
Impact Factor: 1.852

censoring time of a PALT to be $\eta = 5$. For different sample sizes the mean squared error (MSE), Random Absolute Bias (RABias), Relative Error (RE), variance and the approximated two sided confidence limits at 95% and 99% level of significance of the estimators for two sets of parameters are obtained. The data in Table 1 and Table 2 gives the MSE, RABias, RE and variance of the estimators for two sets of parameters $(\theta = 4.50, \beta = 0.30)$ and $(\theta = 4.30, \beta = 0.70)$ respectively. While Table 3 and Table 4 presents the two sided approximate confidence limits at 95% and 99% level of significance for the scale parameter and the acceleration factor.

Table 1: Simulation results for $\theta = 4.50, \beta = 0.30$ at $\eta = 5$

n	Parameters	MSE	RBias	RE	Variance
100	$\hat{\theta}$	0.0062	0.0009	0.0179	0.0062
	$\hat{\beta}$	0.0306	0.0221	0.2187	0.0303
200	$\hat{\theta}$	0.1055	0.0006	0.0738	0.1055
	$\hat{\beta}$	0.0047	0.0228	0.0857	0.0044
300	$\hat{\theta}$	0.0152	0.0014	0.0280	0.0152
	$\hat{\beta}$	0.0024	0.0163	0.0612	0.0022
400	$\hat{\theta}$	0.0131	0.0016	0.0260	0.0131
	$\hat{\beta}$	0.0209	0.0110	0.1807	0.0208
500	$\hat{\theta}$	0.0150	0.0033	0.0278	0.0148
	$\hat{\beta}$	0.0028	0.0048	0.0661	0.0028

Table 2: Simulation results for $\theta = 4.30, \beta = 0.70$ at $\eta = 5$

n	Parameters	MSE	RBias	RE	Variance
100	$\hat{\theta}$	0.0227	0.0053	0.0328	0.0221
	$\hat{\beta}$	0.0002	0.0165	0.0707	0.0002
200	$\hat{\theta}$	0.0399	0.0120	0.0434	0.0369
	$\hat{\beta}$	0.0004	0.0275	0.1000	0.0004
300	$\hat{\theta}$	0.0914	0.0001	0.0657	0.0914
	$\hat{\beta}$	0.0020	0.0040	0.2236	0.0020
400	$\hat{\theta}$	0.0162	0.0070	0.0277	0.0152
	$\hat{\beta}$	0.0002	0.0360	0.0707	0.0001
500	$\hat{\theta}$	0.0266	0.0039	0.0355	0.0263
	$\hat{\beta}$	0.0003	0.0380	0.0066	0.0002

Table 3: Confidence Intervals for $\theta = 4.50, \beta = 0.30$ at $\eta = 5$

n	Parameters	95% LCL	95% UCL	99% LCL	99% UCL
100	$\hat{\theta}$	4.2842	4.8669	4.3318	4.8194
	$\hat{\beta}$	0.1689	0.2244	0.1735	0.2199
200	$\hat{\theta}$	4.2789	5.0319	4.3404	4.9704
	$\hat{\beta}$	0.1663	0.2447	0.1727	0.2383
300	$\hat{\theta}$	4.0079	5.1931	4.1047	5.0963
	$\hat{\beta}$	0.1115	0.2869	0.1259	0.2725
400	$\hat{\theta}$	4.3263	4.8094	4.3656	4.7617
	$\hat{\beta}$	0.1876	0.2268	0.1908	0.2236
500	$\hat{\theta}$	4.3003	4.9361	4.3522	4.8842
	$\hat{\beta}$	0.1647	0.2201	0.1191	0.2657

Table 4: Confidence Intervals for $\theta = 4.30, \beta = 0.70$ at $\eta = 5$

n	Parameters	95% LCL	95% UCL	99% LCL	99% UCL
100	$\hat{\theta}$	4.2498	4.5584	4.2749	4.5332
	$\hat{\beta}$	0.4411	1.1235	0.4968	1.0678
200	$\hat{\theta}$	3.7659	5.0391	3.8698	4.9352
	$\hat{\beta}$	0.6882	0.9482	0.7094	0.9269
300	$\hat{\theta}$	3.6299	5.1581	4.1918	4.5962
	$\hat{\beta}$	0.6951	0.8789	0.7101	0.8639
400	$\hat{\theta}$	4.1749	4.6237	4.2116	4.5870

ISSN: 2277-9655
Impact Factor: 1.852

	$\hat{\beta}$	0.5261	1.0915	0.5723	1.0453
500	$\hat{\theta}$	4.1472	4.6240	4.1861	4.5851
	$\hat{\beta}$	0.7000	0.9075	0.7170	0.8906

Discussion and Conclusion

From above tables it is concluded that for the first set of parameters $(\theta = 4.50, \beta = 0.30)$, the ML estimates have good statistical properties than the second set of parameters $(\theta = 4.30, \beta = 0.70)$ for all sample sizes. Also as the acceleration factor increases the estimates have smaller MSE and RE. As the sample size increases the RABias and MSEs of the estimates of parameters decreases. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimators for the scale parameter and the acceleration factor. When sample size increases, the interval of the estimators decreases. It is also noticed that the intervals of the estimators at $\gamma = 0.95$ is smaller than the interval of estimators at $\gamma = 0.99$.

The results of this simulation study suggests that the ML estimates approximate the true values of the parameters well with good statistical properties and hence it may be say that SSPALT is a suitable model which enables to save time and money considerably without using a high stress to all test units.

Reference

[1] Abd-Elfattah, A. M., A. S. Hassan and S. G. Nasar (2008). “Estimation in step-stress partially accelerated life tests for the Burr type XII distribution using type I censoring”, *Statistical Methodology*, 5, pp. 502-514.

[2] Abdel-Ghaly, A. A., Attia, A. F. and Abdel-Ghani, M. M. (1997). "The Bayesian Estimation of Weibull Parameters in Step Partially Accelerated life Tests with Censored Data". Proceeding of the 32st Annual Conference of Statistics, Computer Sciences and Operation Research, ISSR, Cairo University, 45-59.

[3] Abdel-Ghaly, A. A., Attia, A. F. and Abdel-Ghani, M. M. (2002). "The Maximum Likelihood Estimates in Step Partially Accelerated life Tests for the Weibull Parameters in Censored Data". Communication in Statistics-Theory and Methods, **31**(4), 551-573.

[4] Abdel-Ghani, M. M., (1998). "Investigations of some lifetime Models under Partially Accelerated life Tests". Ph.D. Thesis, Department of Statistics, Faculty of Economics and Political Science, Cairo University, Egypt.

[5] Attia, A. F., Abel-Ghaly, A. A., and Abel-Ghani-, M. M. (1996). "The Estimation Problem of Partially Accelerated Life Tests for Weibull distribution by Maximum likelihood Method with Censored Data". Proceeding of the 31st Annual Conference of Statistics, Computer Sciences and Operation Research, ISSR, Cairo University, 128-138.

[6] Bagdonavicius, V. and Nikulin, M. (2002) “Accelerated life models: Modeling and statistical analysis” Boca Raton: Chapman & Hall.

[7] Bai, D.S., Chung, S.W. (1992). "Optimal Design of Partially Accelerated Life Tests for the Exponential Distribution under Type I Censoring". IEEE Trans. Reliability **41**, 400-406.

[8] Bhattacharyya, G. K. Soejoeti, Z. (1989). "A Tampered Failure Rate Model for Step-Stress Accelerated Life Test". Communication in Statistics-Theory and Methods, 8, 1627-1644.

[9] DeGroot, M. H. and Goel, P. K. (1979). "Bayesian estimation and optimal Design in Partially Accelerated life Testing". Naval Research Logistics Quarterly, **16**(2) 223-235.

[10] Goel, P. K. (1971). "Some Estimation problems in the study of Tampered Random Variables". Ph.D. Thesis Department of Statistics, Cranegie-Mellon University, Pittspurgh, Pennsylvania.

[11] Ismial, A. A. (2006). "On the Optimal Design of Step-Stress Partially Accelerated Life Tests for the Gompertz Distribution with Type I Censoring". InterStat, Elctronic Journal. http:/interstat.stat.vt.edu/interstat/Articles/No2006.pdf.

[12] Mann, N. R. and Singpurwalla, N.D. (1983) “Life Testing Encyclopedia of statistical sciences: USA: Wiley, Vol. 4, pp. 632–639.

[13] Meeker, W. Q. and Escobar, L. A. (1998) “Statistical methods for reliability data” New York: Wiley.

[14] Nelson, W. B. (1980). Accelerated life testing: step-stress models and data analyses. IEEE Transactions on Reliability, 29, 103–108.

[15] Nelson, W. B. (1990) “Accelerated testing: Statistical models, test plans and data analyses” New York: Wiley.